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This booklet, one of a series, has been developed for the project, A Program for Mathematically Underdeveloped Pupils. A project team, including inservice teachers, is being used to write and develop the materials for this program. The materials developed in this booklet include (1) the addition and subtraction of whole numbers on the number line, (2) the addition and subtraction of integers on the number line, and (3) the idea of inequality. Accompanying these booklets will be a "Teaching Strategy Booklet" which will include a description of teacher techniques, methods, suggested sequences, academic games, and suggested visual materials. (RP)

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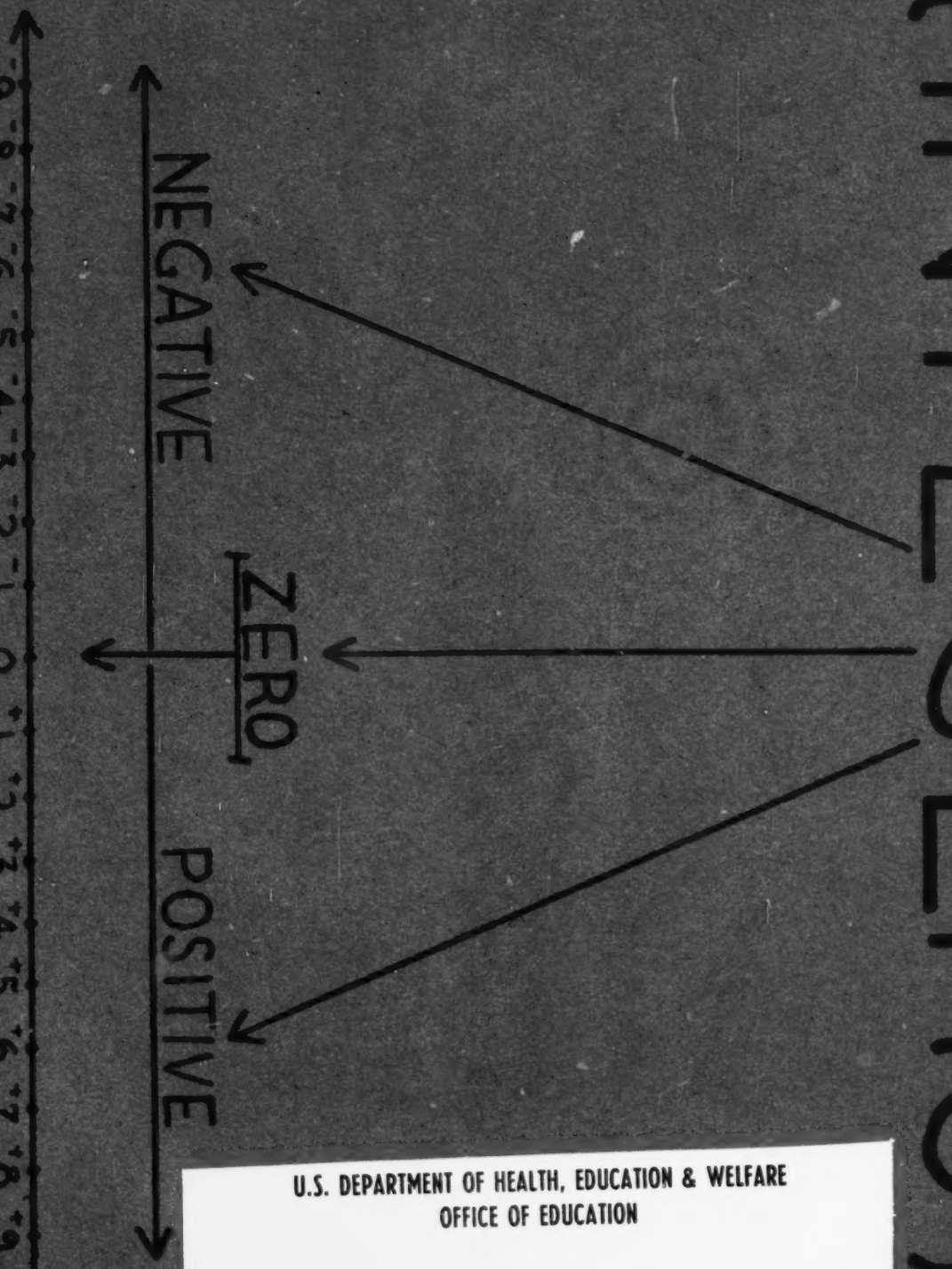
INTEGERS; ADDITION AND SUBTRACTION, "ACTION".



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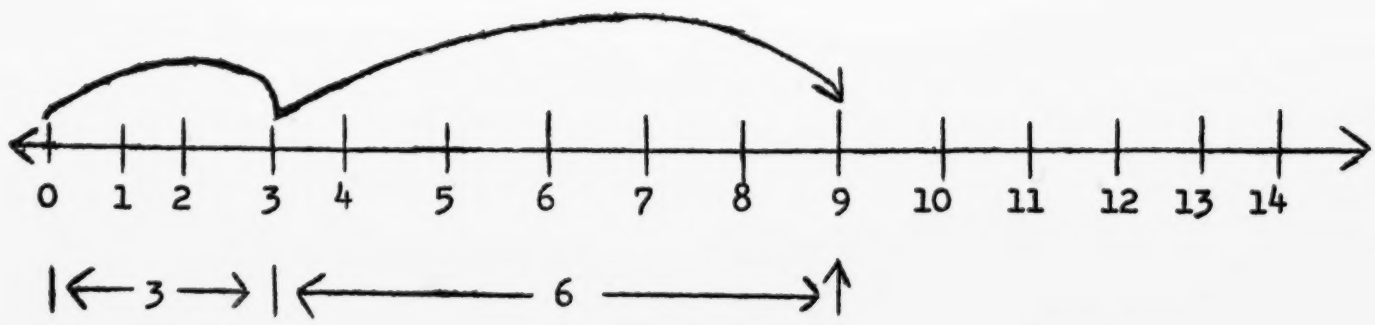
A number line is often used to illustrate number relationships and number operations. For example, the operations of addition and subtraction can be shown as follows:

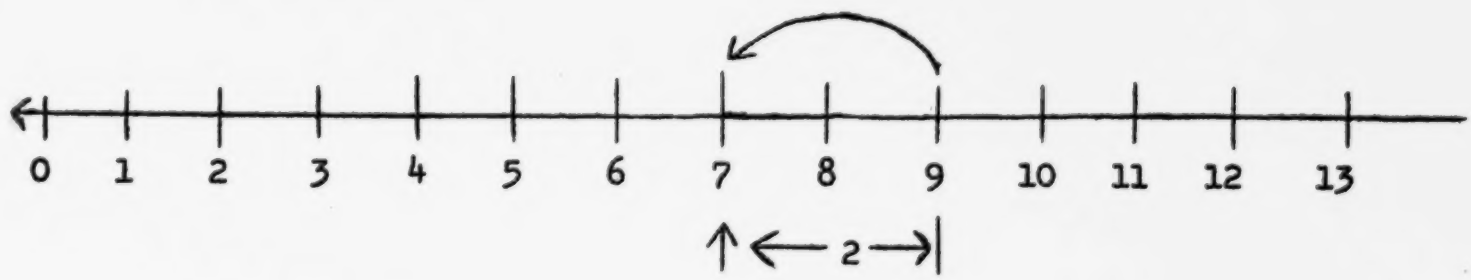
$$3 + 6 = m$$



or $3 + 6 = 9$

To show this, first move three spaces and then six more spaces (forward or to the right). For subtraction:

$$9 - 2 = m$$



or $9 - 2 = 7$

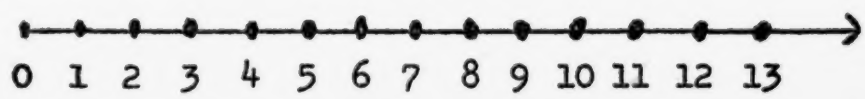
We move nine spaces and then back two spaces (to the left) or simply start at nine and move back two spaces.

Activities

Illustrate each of the addition or subtraction problems on the number line. Supply the missing value.

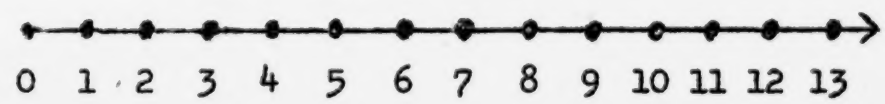
a.) 5 + 4 = m

m =



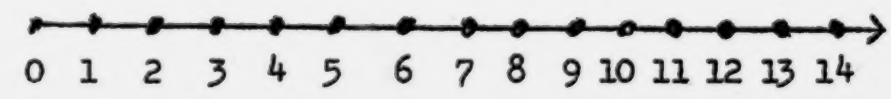
b.) 7 + 6 = m

m =



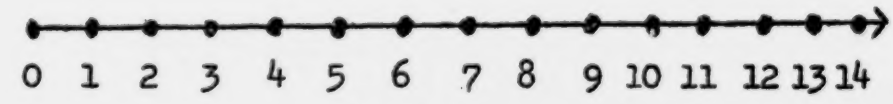
c.) 9 + 4 = m

m =



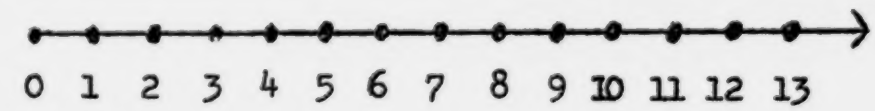
d.) 8 + 6 = m

m =



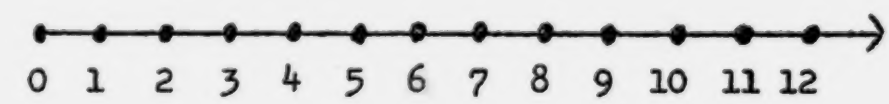
e.) 7 - 5 = m

m =



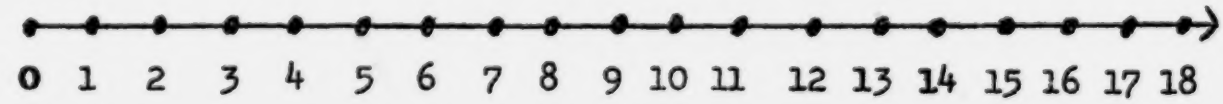
f.) 2 + 9 = m

m =



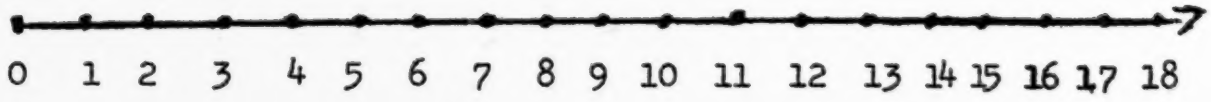
g.) 9 - 8 = m

m =



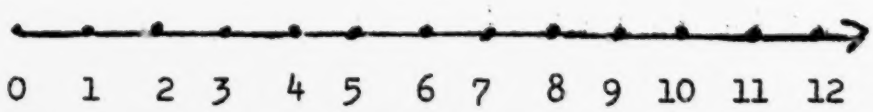
h.) 8 - 8 = m

m =



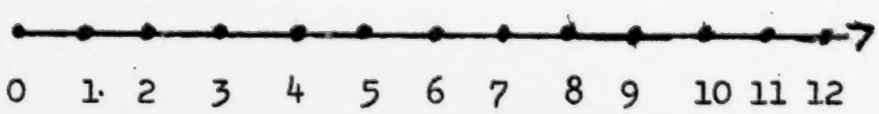
i.) 2 + 6 = m

m =



j.) 0 + 4 = m

m =



2. **Show each of these on the number line.**

a.) 2 + 3 = m

b.) 3 + 2 = m

c.) 5 + 2 = m

d.) 2 + 5 = m

What do you notice about your answers to a and b?

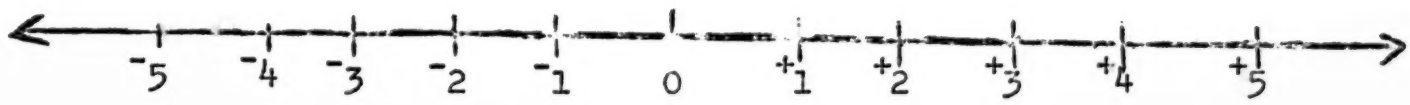
What do you notice about your answers to c and d?

If two numbers are to be added, the <u>commutative property</u> of addition says you can change the order of the two numbers without changing the answer. Do you think this is true for any two numbers?

The numbers we have been working with are called the whole numbers; they begin with zero and continue to "add" one to reach the next whole number:

whole numbers - {0, 1, 2, 3, 4, ...}

Let's examine a new set of numbers. Each point below represents one of the new numbers. They are called the integers.



These numbers continue on both to the "left" and "right" of zero. They can also be shown as:

integers - $\{ \ldots {}^{-}3, {}^{-}2, {}^{-}1, 0, {}^{+}1, {}^{+}2, {}^{+}3, \ldots \}$

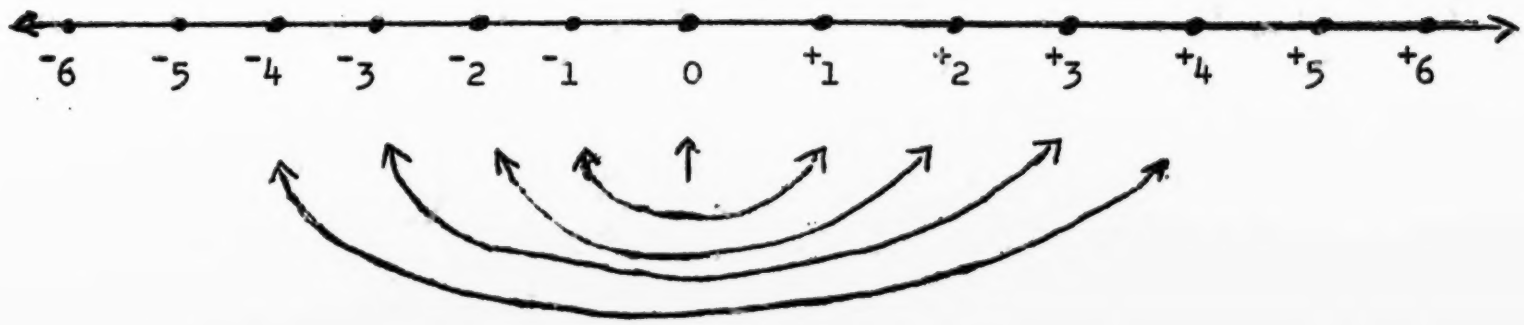
These numbers are sometimes called "signed" numbers or "directed" numbers. The numbers to the left of zero are called negative integers and to the right of zero are called positive integers. The positive and negative symbols are placed above and just to the left of each integer (zero is neither positive nor negative). The numbers are read as:

positive five = ${}^{+}5$
negative three = ${}^{-}3$

Certain "pairs" of integers are called opposites. Below are some examples of pairs of opposites:

${}^{-}5$ and ${}^{+}5$, ${}^{+}2$ and ${}^{-}2$, ${}^{+}3$ and ${}^{-}3$

What do you notice about opposites? Look at some of the opposites shown at the top of the next page.



Are opposites the same number of spaces from zero? Does zero have an opposite? Zero is called its own opposite. This makes every integer have an opposite.

Activities

1. Supply the missing opposites.

a.) ___, $^{+}7$

b.) $^{-}3$, ___

c.) $^{-}17$, ___

d.) $^{+}25$, ___

e.) 0, ___

f.) $^{-}86$, ___

g.) ___, $^{+}1$

h.) $^{-}42$, ___

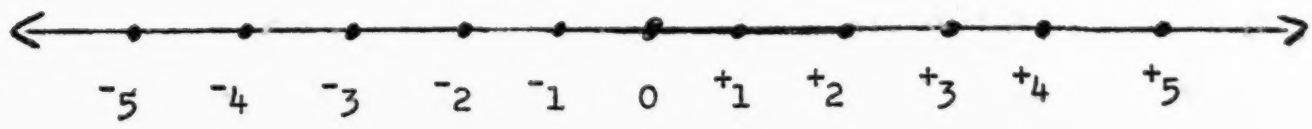
2. Supply the missing consecutive integers.

$\underline{^{-}5}$, $\underline{^{-}4}$, ___, ___, ___ $\underline{0}$, ___, ___, ___, ___

3. Counting by 2's, supply the missing integers.

___, ___, $\underline{^{-}4}$, $\underline{^{-}2}$, ___, ___, $\underline{^{+}4}$, $\underline{^{+}6}$

On the number line, a number to the "right of" another number is the greater of the two numbers. For example:



$^{+}3$ is to the right of $^{-}2$

then $^{+}3$ "is greater than" $^{-}2$

Remember the relation symbols "is greater than" and "is less than".

Symbol	meaning
$>$	"is greater than"
$<$	"is less than"

Now we can write:

$$^{+}3 > {}^{-}2$$

Activities

1. Place an "is greater than symbol ($>$)" or an "is less than symbol ($<$)" between each pair of integers (make it a true statement).

a.) $^{+}3$ ___ $^{+}4$

b.) $^{-}7$ ___ $^{+}9$

c.) $^{+}5$ ___ $^{-}5$

d.) $^{-}8$ ___ $^{-}7$

e.) $^{-}6$ ___ $^{+}3$

f.) 0 ___ $^{-}100$

2. Rearrange the following integers so that they will be in order from the smallest to the largest. Place them in the space provided.

a.) $\{0, {}^{-}8, {}^{+}4, {}^{-}2, {}^{-}9, {}^{+}6\}$ ------> { }

b.) $\{{}^{+}6, {}^{+}8, 0, {}^{-}2, {}^{+}3, {}^{-}8\}$ ------> { }

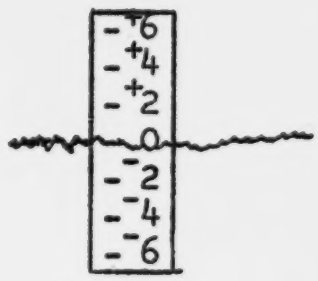
c.) $\{{}^{-}7, {}^{+}5, 0, {}^{+}1, {}^{+}6, {}^{-}1\}$ ------> { }

d.) $\{{}^{+}10, {}^{-}10, {}^{+}8, {}^{+}4, 0, {}^{+}3\}$ ------> { }

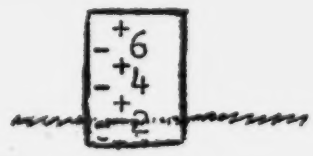
e.) $\{{}^{-}6, {}^{+}7, {}^{-}5, 0, {}^{+}2, {}^{-}1\}$ ------> { }

f.) $\{{}^{+}9, {}^{+}6, {}^{-}3, 0, 2, 1\}$ ------> { }

3. Mr. Black has a lake behind his house. He likes to keep track of the water level in the lake so that he can tell when the fishing is good. During one winter Mr. Black placed a stake in the middle of the lake. It looked like the picture below:

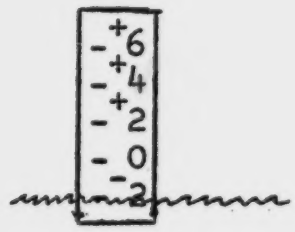


In April Mr. Black noticed that the stake looked like this:



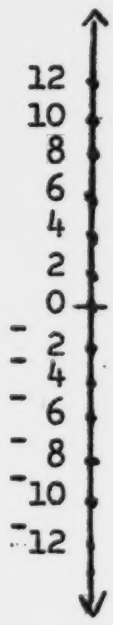
What had happened to the water level in the lake?

In August Mr. Black's stake looked like this:



What has happened to the water level in the lake?

4.



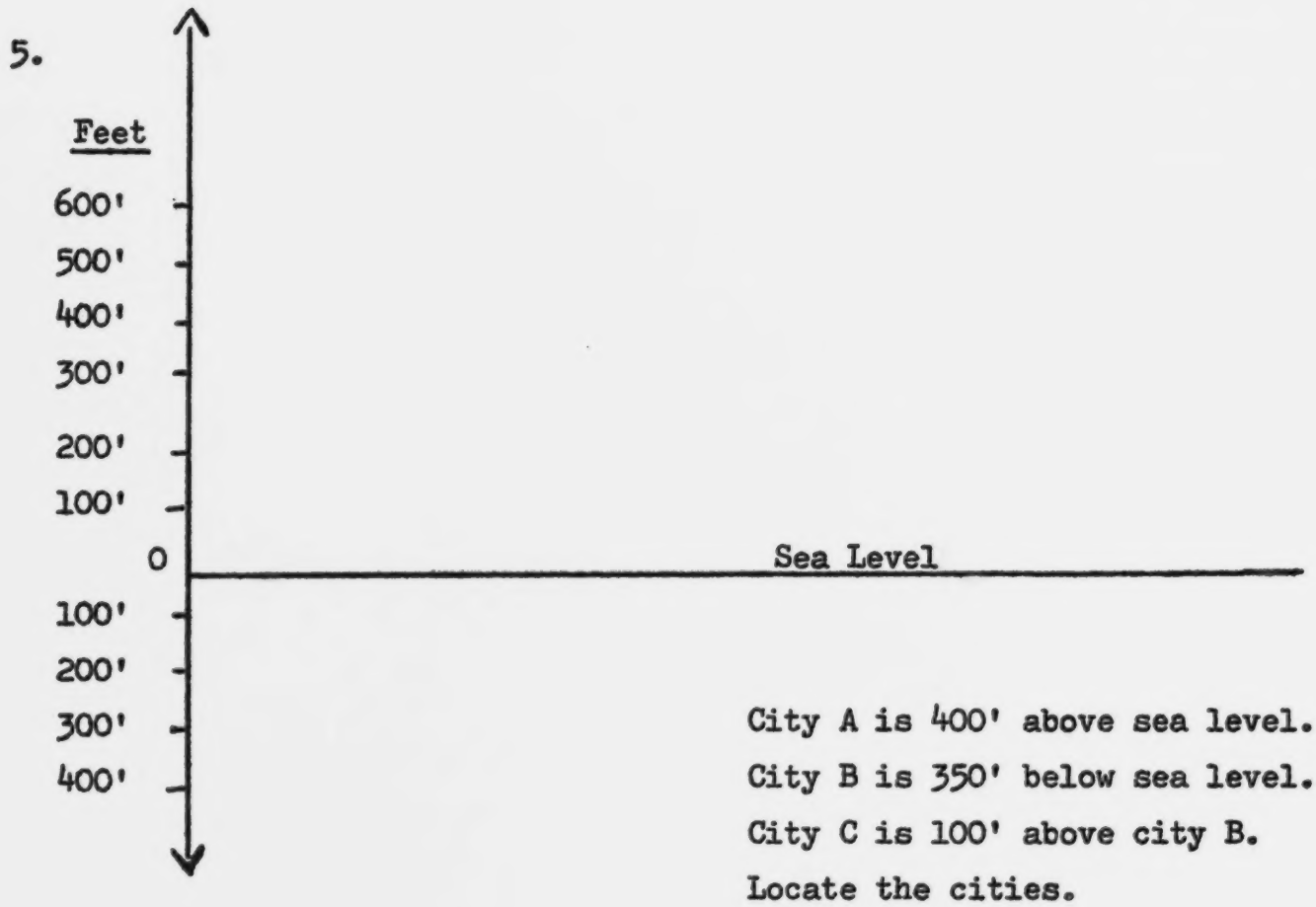
City A has a temperature of 10° below zero.

City B has a temperature of 3° below zero.

City C fell 5° from 3° below zero.

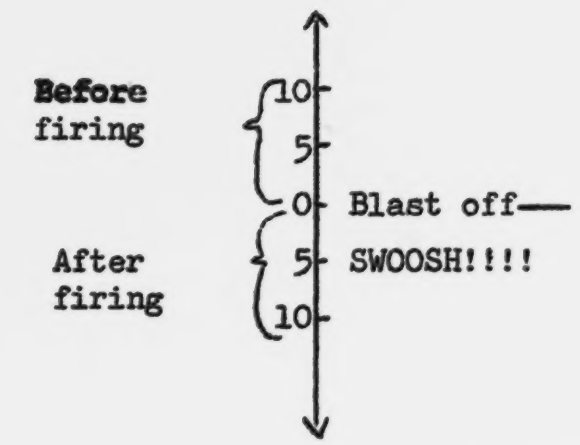
Locate the cities on the number line.

5.



City A is 400' above sea level.
City B is 350' below sea level.
City C is 100' above city B.
Locate the cities.

6. Rocket Firing:



How long is it from 10 seconds <u>before</u> firing to 3 seconds after firing?

How long is it from 7 seconds <u>after</u> firing to 10 seconds after firing?

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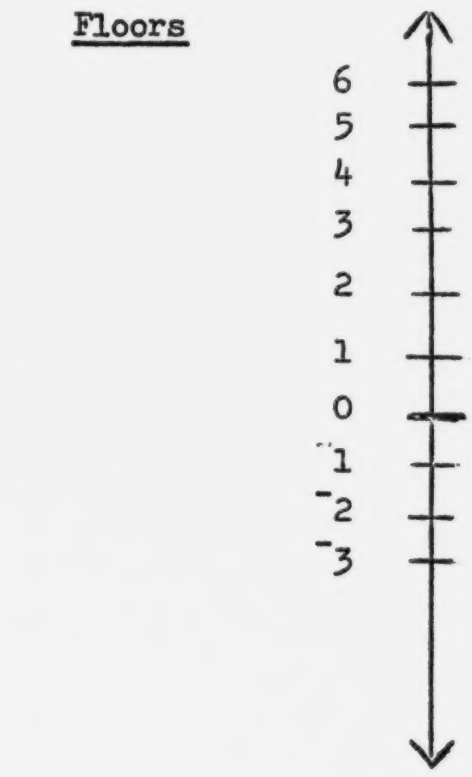
* * * 7 seconds <u>after</u> firing is how many seconds <u>after</u> 4 seconds before firing?

7.

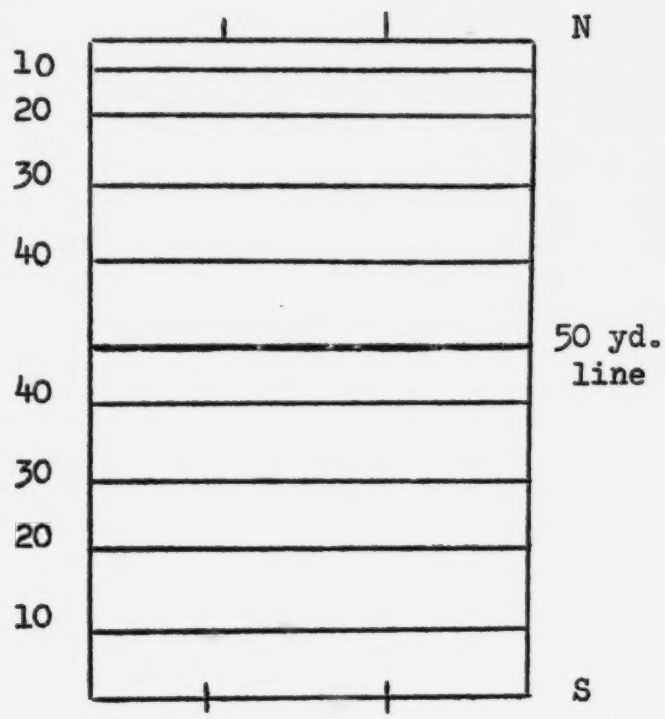


(12) No. Line

This no. line represents the floors in a building. The ground floor is shown by zero. The basement levels are represented by negative integers.

If an elevator starts in the second basement goes up 5 flights and down two, where would it stop?

8.



This represents a football field.

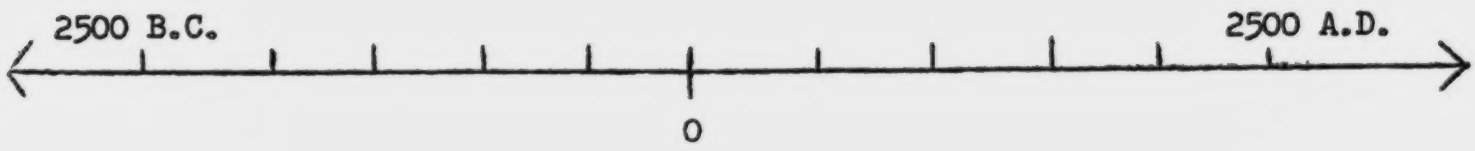
Player A starts at 50 yard line going N and has a loss of 4 yards. Where would he be?

Player B starts at the 30 yard line going south and has a gain of 20 yards; then a loss of 10. Where would he be?

Player C starting at 50 yards gains 20 going N, and then loses 20. Where would he be?

9. (13) No. line

The concept of time can be shown on a no. line. The birth of Christ would be shown at zero. B.C. to the left and A.D. to the right.



Show the landing of Columbus----------

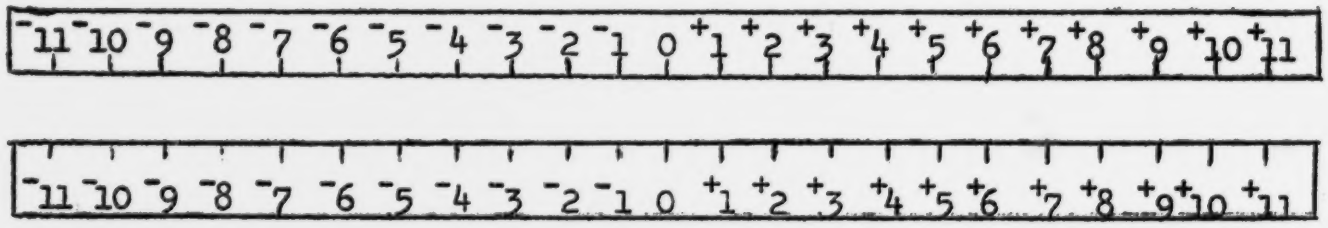
Show <u>your</u> birthday----------

Operations--Addition and Subtraction

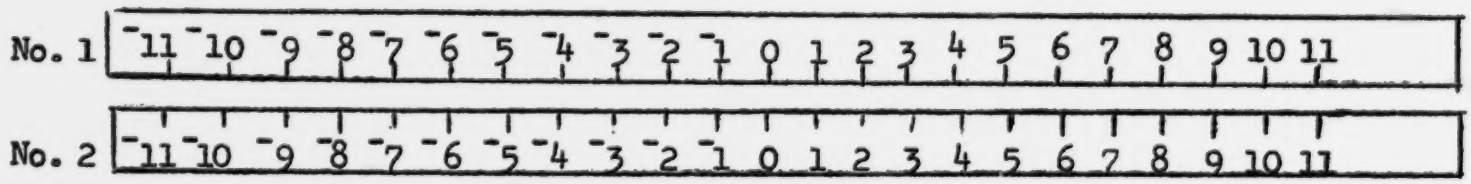
Follow the activity below as an introduction to adding and subtracting integers.

Activities

1. Cut out two long strips of paper and number them like the examples below (this will work for integers $^{-}11$ to $^{+}11$).

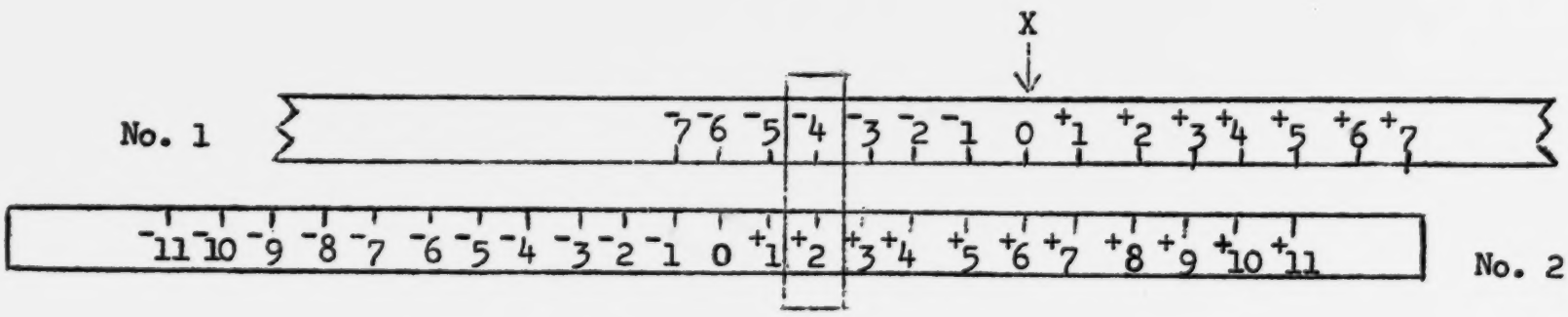


You have made what we shall call a "linear slide rule". This slide rule will add or subtract integers for you. Let's see how it works for addition.



Slide number 1. moves. Slide number 2. stays in a stationary position.

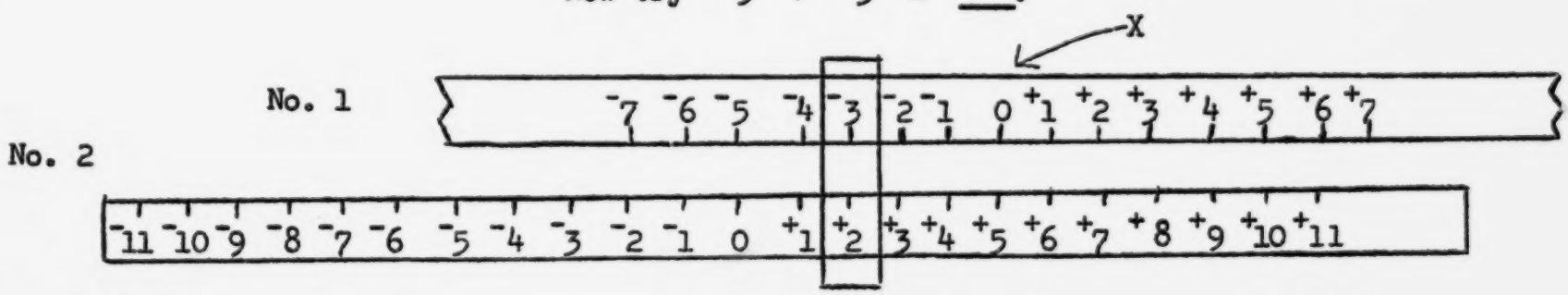
Consider the problem: $^{+}6 + {}^{-}4 =$ ___.



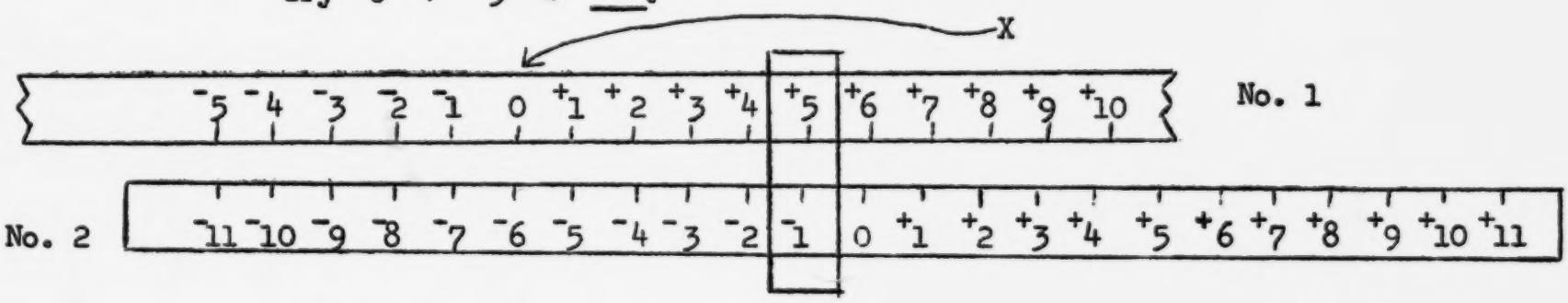
1. Slide number 2 stays still and we move slide number 1. so that the zero on slide number 1. is directly over the first number in our problem (in this case $^{+}6$).
2. Now find the second number in our problem ($^{-}4$) on slide number 1. The answer to our problem is on slide number 2. under $^{-}4$. What is the answer?

Yes, $^{+}6 + {}^{-}4 = {}^{+}2$.

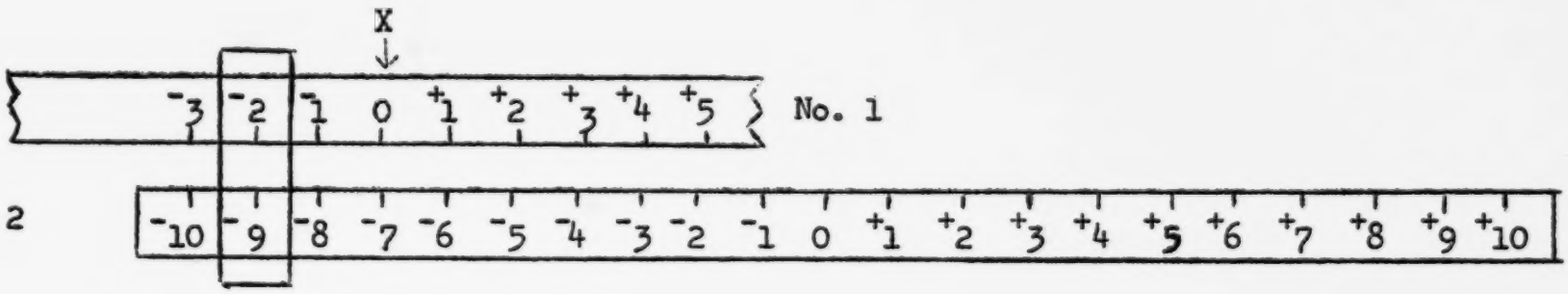
Now try $^{+}5 + {}^{-}3 =$ ___.



Try $^{-}6 + {}^{+}5 =$ ___.



And $^{-}7 + {}^{-}2 =$ ___.



2. Work the following addition problems with your "linear slide rule".

a.) $^{+}7 + {}^{-}4 =$ ___.

b.) $^{+}5 + {}^{-}6 =$ ___.

c.) $^{+}3 + {}^{-}6 =$ ___.

d.) $^{-}2 + {}^{-}5 =$ ___.

e.) $^{-}4 + {}^{-}3 =$ ___.

f.) $^{-}9 + {}^{-}1 =$ ___.

g.) $^{-}4 + {}^{+}6 =$ ___.

h.) $^{-}8 + {}^{+}4 =$ ___.

i.) $^{-}2 + {}^{+}7 =$ ___.

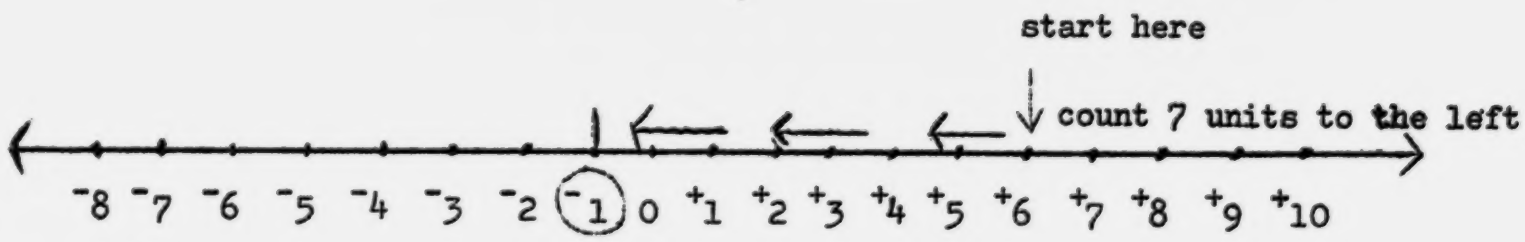
j.) $^{+}5 + {}^{+}4 =$ ___.

k.) $^{+}6 + {}^{+}4 =$ ___.

l.) $^{+}3 + {}^{+}1 =$ ___.

You can also add integers by simply counting spaces on the number line. Here is how you do it.

Example: $^{+}6 + {}^{-}7 =$ ___.

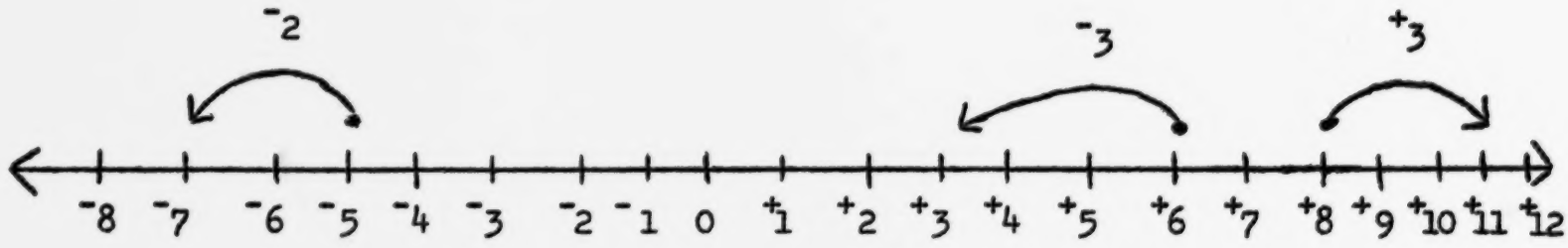


Start with the first number in your problem. Locate 6 on the number line and then count 7 units to the left. If the 7 had been a positive number, you would have counted 7 units to the right instead of 7 units to the left. <u>When you are adding a positive number you count to the right</u> and <u>when you are adding a negative number you count to the left</u>.

"Moves"--Positive and Negative

Adding integers can be shown by "moves" on a number line. The line below has three moves shown. The three problems are:

1. From $^{+}6$, make a move of $^{-}3$.
2. From $^{+}8$, make a move of $^{+}3$.
3. From $^{-}5$, make a move of $^{-}2$.



These moves illustrate:

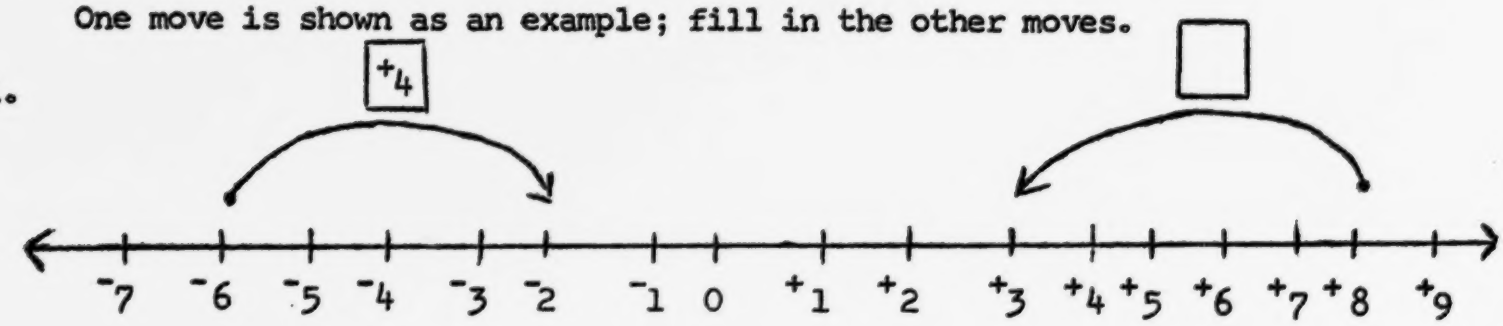
1. $^{+}6 + {}^{-}3 = {}^{+}3$
2. $^{+}8 + {}^{+}3 = {}^{+}11$
3. $^{-}5 + {}^{-}2 = {}^{-}7$

Activities

Notice that a move to the left is a negative move and a move to the right is a positive move.

One move is shown as an example; fill in the other moves.

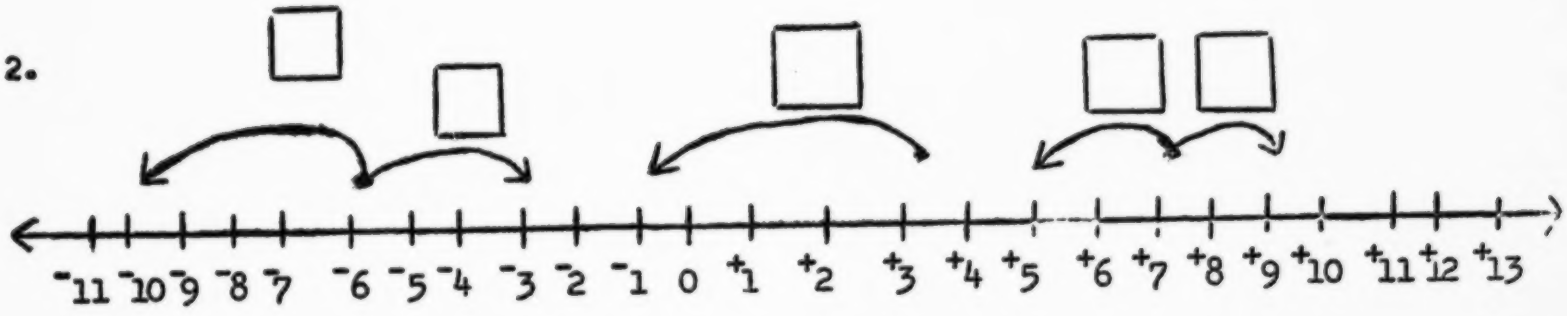
1.



$^{-}6 + \boxed{^{+}4} = {}^{-}2$

$^{+}8 + \square = {}^{+}3$

2.



$^{-}6 + \square = {}^{-}10$

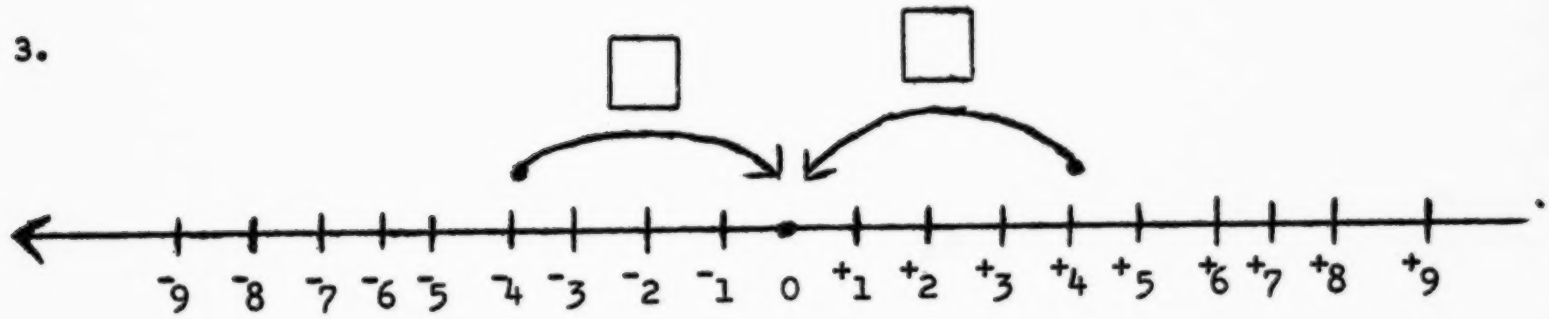
$^{-}6 + \square = {}^{-}3$

$^{+}3 + \square = {}^{-}1$

$^{+}7 + \square = {}^{+}5$

$^{+}7 + \square = {}^{+}9$

3.



$^{-}4 + \square = 0$

$^{+}4 + \square = 0$

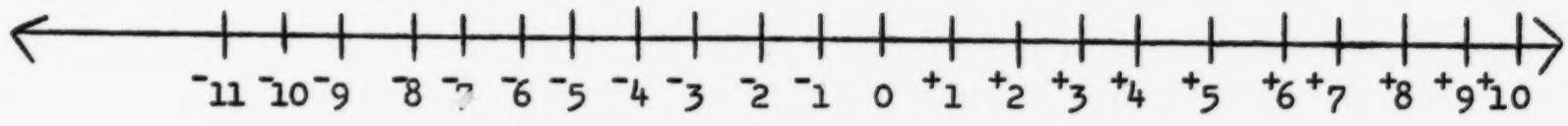
4. On the number line at the top of page 17, make the following moves.

A. From $^{+}6$, move $^{+}2$.

B. From $^{-}7$, move $^{-}3$.

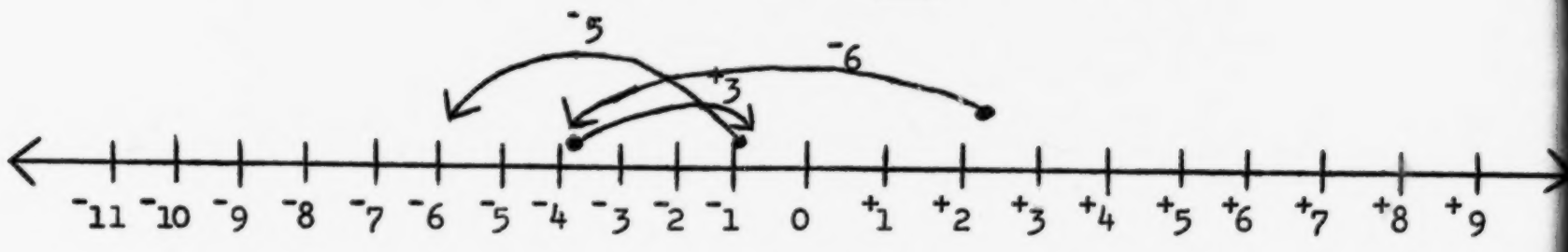
C. From $^{-}3$, move $^{+}3$.

D. From $^{+}2$, move $^{-}2$.



5. The problem below can be shown by "multiple" moves (more than <u>one</u> move).

$$^{+}2 + ^{-}6 + ^{+}3 + ^{-}5 = \boxed{^{-}6}$$

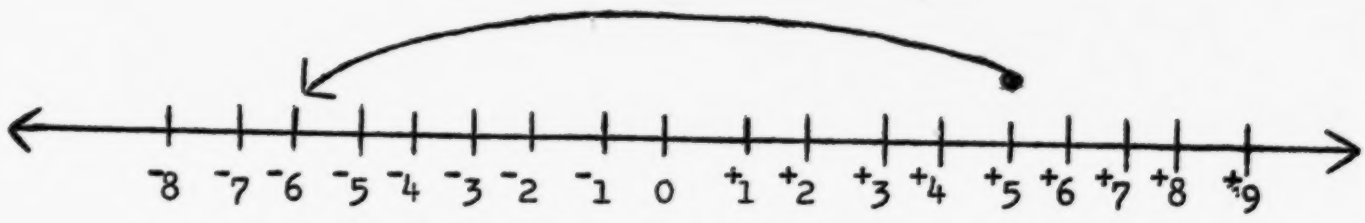


An easier way would be to combine the positive and then combine the negatives--then only one move can be shown.

$$^{+}2 + ^{-}6 + ^{+}3 + ^{-}5 =$$

$$(^{+}2 + ^{+}3) + (^{-}6 + ^{-}5) =$$

$$^{+}5 + ^{-}11 = \boxed{^{-}6}$$



Without the number line, solve:

a. $^{+}3 + ^{-}6 + ^{+}2 + ^{-}1 = \square$

b. $^{-}5 + ^{-}2 + ^{+}2 + ^{+}5 = \square$

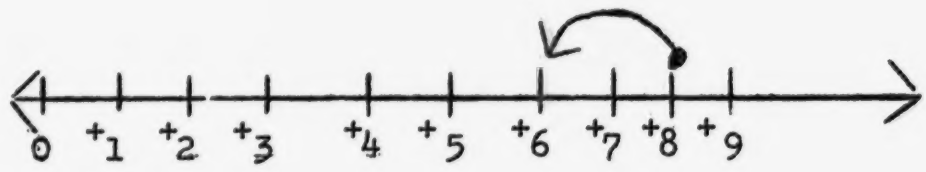
c. $^{+}8 + ^{-}8 + ^{+}3 + ^{+}3 = \square$

d. $^{+}16 + ^{-}10 + ^{-}21 + ^{+}18 + ^{-}16 = \square$

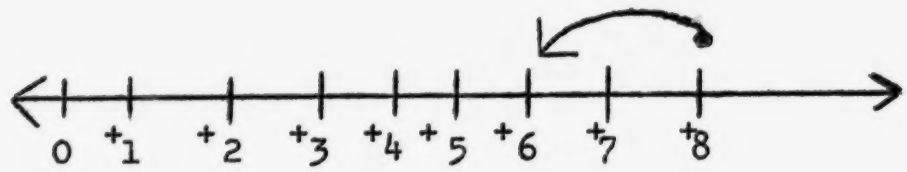
Subtraction—Adding Opposites

Compare the first four pairs of problems below.

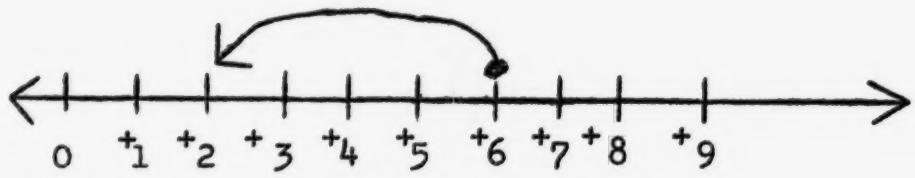
1A. $^{+}8 - {}^{+}2 = {}^{+}6$



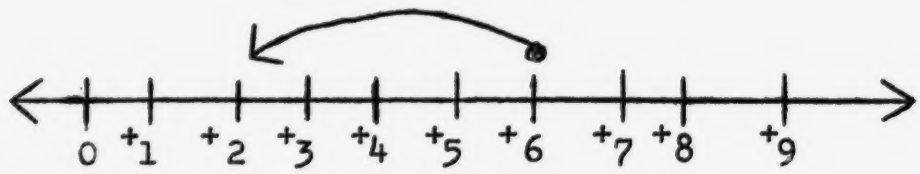
1B. $^{+}8 + {}^{-}2 = {}^{+}6$



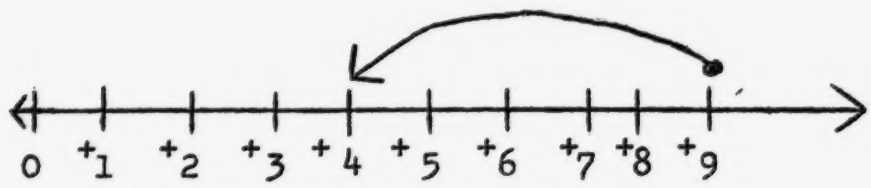
2A. $^{+}6 - {}^{+}4 = {}^{+}2$



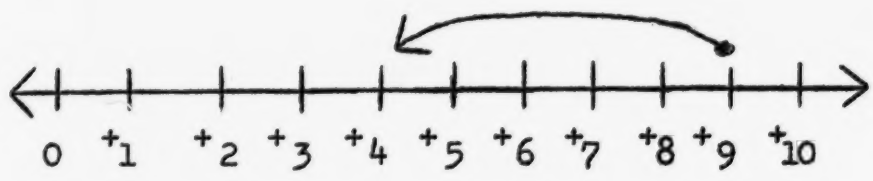
2B. $^{+}6 + {}^{-}4 = {}^{+}2$



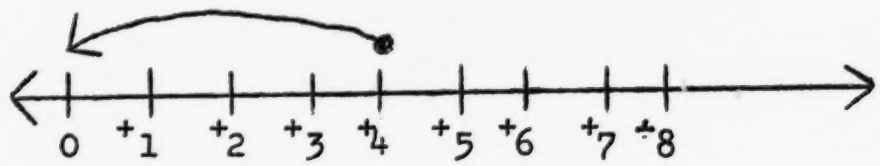
3A. $^{+}9 - {}^{+}5 = {}^{+}4$



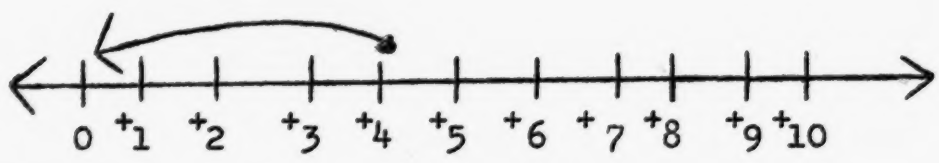
3B. $^{+}9 + {}^{-}5 = {}^{+}4$



4A. $^{+}4 - {}^{+}4 = 0$



4B. $^{+}4 + {}^{-}4 = 0$



To do subtraction, simply add an opposite.

Since you have learned to add integers, you can solve any subtraction by changing it to an addition problem that gives the same answer. The idea to do subtraction is to add opposites. See the examples below.

$$\begin{bmatrix} {}^{+}6 - \overset{\downarrow}{\underset{\downarrow}{\oplus 5}} = {}^{+}1 \\ {}^{+}6 + \ominus 5 = {}^{+}1 \end{bmatrix}$$

$$\begin{bmatrix} {}^{+}5 - \overset{\downarrow}{\underset{\downarrow}{\ominus 3}} = {}^{+}8 \\ {}^{+}5 + \oplus 3 = {}^{+}8 \end{bmatrix}$$

$$\begin{bmatrix} {}^{-}6 - {}^{+}3 = {}^{-}9 \\ {}^{-}6 + {}^{-}3 = {}^{-}9 \end{bmatrix}$$

Activities

Solve the following subtraction problems by adding opposites of the numbers to be subtracted. Check each by using your linear slide rule.

1. $^{-}9 - {}^{+}7 =$ _____
2. $^{+}7 - {}^{-}8 =$ _____
3. $^{+}8 - {}^{+}9 =$ _____
4. $^{+}9 - {}^{-}2 =$ _____
5. $^{-}9 - {}^{-}6 =$ _____
6. $^{+}10 - {}^{+}4 =$ _____
7. $^{-}26 - {}^{+}14 =$ _____
8. $^{-}6 - {}^{-}6 =$ _____
9. $^{+}8 - {}^{+}8 =$ _____
10. $^{-}12 - {}^{+}12 =$ _____
11. $^{+}9 - {}^{-}9 =$ _____
12. $^{+}6 - {}^{+}6 =$ _____

A problem can have several terms. The problem below can be rewritten in terms of addition and then solved very easily.

$$^{+}6 + {}^{-}2 - {}^{+}7 - {}^{-}14 = ____$$

$$^{+}6 + {}^{-}2 + {}^{-}7 + {}^{+}14 = ____$$

(group: like terms) $(^{+}6 + {}^{+}14) + (^{-}2 + {}^{-}7) = ____$

$$^{+}20 + {}^{-}9 = {}^{+}11$$

Activities

Solve each by adding opposites.

1. $^{+}6 - {}^{-}2 + {}^{+}6 - {}^{-}10 = ____$

2. $^{+}15 - {}^{-}7 - {}^{-}1 - {}^{+}8 = ____$

3. $^{-}9 - {}^{-}2 - {}^{-}7 + {}^{+}9 = ____$

4. $^{+}18 - {}^{+}18 - {}^{+}6 + {}^{+}6 = ____$

5. $^{+}24 - {}^{-}24 + {}^{+}24 - {}^{-}24 = ____$